The CIA (consistency in aggregation) approach

A new economic approach to elementary indices

Dr Jens Mehrhoff*, Head of Section Business Cycle and Structural Economic Statistics

Outline

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"Elementary, my dear Watson!" (Sherlock Holmes)

1. Motivation

– Practical **consumer price indices are constructed in two stages**:

1. **a first stage at the lowest level of aggregation** where price information is available but associated expenditure or **quantity information is not available** and
2. **a second stage of aggregation** where **expenditure information is available at a higher level of aggregation**.

– Although the scope of the discussion is on **the choice of the index formula at the elementary level**, this choice eventually **depends on the target price index** at the aggregate level.

– HICP is not intended to measure the cost of living (COLI), rather, it is **a cost of goods index (COGI)**.

2. Background
Bilateral price indices

- We specify **two accounting periods**, $t \in \{0, 1\}$, for which we have micro price and quantity data for n commodities (bilateral index context).
- Denote the **price and quantity** of commodity $i \in \{1, \ldots, n\}$ in period t by p_i^t and q_i^t, respectively.

- A very simple approach to the determination of a price index over a group of commodities is **the (fixed) basket approach**.
- Define the **Lowe** (1823) **price index**, P_{Lo}, as follows:
- $$P_{Lo} = \frac{\sum_{i=1}^{n} p_i^1 \cdot q_i}{\sum_{i=1}^{n} p_i^0 \cdot q_i}.$$

- There are **two natural choices** for the reference basket:
  - the period 0 commodity vector $\boldsymbol{q}^0 = (q_1^0, \ldots, q_n^0)$ or
  - the period 1 commodity vector $\boldsymbol{q}^1 = (q_1^1, \ldots, q_n^1)$.

2. Background
Laspeyres, Paasche and Fisher

– These two choices lead to
  - the **Laspeyres** (1871) **price index** P_L, if we choose $\boldsymbol{q} = \boldsymbol{q^0}$, and
  - the **Paasche** (1874) **price index** P_P, if we choose $\boldsymbol{q} = \boldsymbol{q^1}$:

– $$P_L = \frac{\sum_{i=1}^{n} p_i^1 \cdot q_i^0}{\sum_{i=1}^{n} p_i^0 \cdot q_i^0},\ P_P = \frac{\sum_{i=1}^{n} p_i^1 \cdot q_i^1}{\sum_{i=1}^{n} p_i^0 \cdot q_i^1}.$$

– According to the CPI Manual (ILO et al., 2004), "**the Paasche and Laspeyres price indices are equally plausible**."

– Taking an evenly weighted average of these basket price indices leads to **symmetric averages**.

– **The geometric mean**, which leads to the **Fisher** (1922) **price index**, P_F, is defined as:

– $$P_F = \sqrt{P_L \cdot P_P}\ .$$

2. Background
Keynes' pure theory of money

- In his 1930 *A Treatise on Money* (pp. 95-120), Keynes deals with the theory of **comparisons of purchasing power**.
- Comparisons of purchasing power mean comparisons of the command of money over two collections of commodities which are in some sense "equivalent" to one another, and **not over quantities of utility**.

- Applying **the "method of limits"** establishes that **in any case the measure of the change in the value of money lies between the Laspeyres and Paasche price indices**.

- **The "crossing of formulae"**, to which Fisher has devoted much attention, is, in effect, **an attempt to carry the method of limits somewhat further – further** (in Keynes' opinion) **than is legitimate**.

- We can concoct **all sorts of algebraic function of P_L and P_P**, and **there will not be a penny to choose between them**.

2. Background
Elementary indices

- Suppose that there are ***M* lowest-level items or specific commodities** in a chosen elementary category.
- Denote the period *t* **price** of item *m* by p_m^t for $t \in \{0, 1\}$ and for items $m \in \{1, \ldots, M\}$.
- The **Dutot** (1738) **elementary price index**, P_D, is equal to the *arithmetic* average of the *M* period 1 prices divided by the *arithmetic* average of the *M* period 0 prices.
- The **Carli** (1764) **elementary price index**, P_C, is equal to the *arithmetic* average of the *M* item price ratios or price relatives, p_m^1/p_m^0.
- The **Jevons** (1865) **elementary price index**, P_J, is equal to the *geometric* average of the *M* item price ratios or price relatives, p_m^1/p_m^0, or the *geometric* average of the *M* period 1 prices divided by the *geometric* average of the *M* period 0 prices.
- $P_D = \frac{\frac{1}{M}\sum_{m=1}^{M} p_m^1}{\frac{1}{M}\sum_{m=1}^{M} p_m^0}, \; P_C = \frac{1}{M}\sum_{m=1}^{M}\frac{p_m^1}{p_m^0}, \; P_J = \sqrt[M]{\prod_{m=1}^{M}\frac{p_m^1}{p_m^0}} = \frac{\sqrt[M]{\prod_{m=1}^{M} p_m^1}}{\sqrt[M]{\prod_{m=1}^{M} p_m^0}}.$

2. Background
Is the Carli index really "upward biased"?

- The sole argument frequently put forward why the Carli index should be abandoned, is **the claim that it has an "upward bias"** with reference to the time reversal test or circularity test (cf. Diewert, 2012):
- $P_C(\boldsymbol{p^0}, \boldsymbol{p^1}) \cdot P_C(\boldsymbol{p^1}, \boldsymbol{p^2}) = P_C(\boldsymbol{p^0}, \boldsymbol{p^1}) \cdot P_C(\boldsymbol{p^1}, \boldsymbol{p^0}) \geq 1 = P_C(\boldsymbol{p^0}, \boldsymbol{p^0})$ for $\boldsymbol{p^2} = \boldsymbol{p^0}$.

- But **this argument is useless in the bilateral index context** where we can compare the two periods under consideration directly, i.e. there is no bias at all:
- $P_C(\boldsymbol{p^0}, \boldsymbol{p^2}) = P_C(\boldsymbol{p^0}, \boldsymbol{p^0}) = 1$ for $\boldsymbol{p^2} = \boldsymbol{p^0}$.

- In the context of chain indices, **the elementary aggregates only feed into the higher-level indices** in which the elementary price indices – comparing periods *t*-1 and *t* (!) – are averaged using a set of pre-determined weights (chain indices are non-aggregable); **the Dutot, Carli and Jevons indices are, thus, not chain-linked**.
- What is more, it apparently fell into oblivion that **the then chain-linked Laspeyres, Paasche, Fisher, Walsh and Törnqvist price indices are subject to chain drift**; i.e. all chain indices are path dependent, which is the opposite of transitivity.

3. Economic approach

- The CPI Manual, paragraphs 20.71-20.86, has a section in it which describes **an economic approach to elementary indices**.

- This section has sometimes been used to **justify the use of the Jevons index**, i.e. the geometric mean, **over the use of the Carli index**, i.e. the arithmetic mean, or vice versa **depending on how much substitutability exists** between items within an elementary stratum.

- **This is a misinterpretation of the analysis** that is presented in this section of the Manual.

- **Thus, the economic approach cannot be applied at the elementary level unless price and quantity information are both available.**
- ***Such information is typically not available***, which is exactly the reason elementary indices are used rather than target indices. (Diewert, 2012, "Consumer Price Statistics in the UK")

4. Consistency approach
Consistency in aggregation

- **The consistency in aggregation (CIA) approach** newly developed (Mehrhoff, 2010, Jahr. Nationalökon. Statist.) fills the void of **guiding the choice of the elementary index** (for which weights are not available) **that corresponds to the characteristics of the index at the second stage** (where weights are actually available).
- It contributes to the literature by looking at how **numerical equivalence between an unweighted elementary index and a weighted aggregate index** can be achieved, **independent of the axiomatic properties**.

- *Consistency in aggregation* means that **if an index is calculated stepwise by aggregating lower-level indices** to obtain indices at progressively higher levels of aggregation, **the same overall result** should be obtained **as if the calculation had been made in one step**.

4. Consistency approach
Cost of goods index

- To reiterate, **we measure the change in the cost of purchasing a fixed basket of goods and services**, and *not* the change in the minimum cost of maintaining a given level of utility or welfare.

- The use of **the Dutot and Carli formulae** at the elementary level of aggregation for *homogeneous* items can be perfectly **consistent with a Laspeyres index concept**.

- **The Laspeyres price index** can be rewritten in an alternative manner as follows:

- $$P_L = \frac{\sum_{m=1}^{M} p_m^1 \cdot q_m^0}{\sum_{m=1}^{M} p_m^0 \cdot q_m^0} = \sum_{m=1}^{M} \frac{p_m^1}{p_m^0} \cdot \frac{p_m^0 \cdot q_m^0}{\sum_{l=1}^{M} p_l^0 \cdot q_l^0} = \sum_{m=1}^{M} \frac{p_m^1}{p_m^0} \cdot s_m^0,$$

- where s_m^0 is the **period 0 expenditure share** on commodity *m*.

4. Consistency approach
A thought experiment

- The first case is where the underlying preferences are **Leontief preferences**, i.e. **consumers prefer not to make any substitutions** in response to changes in relative prices (zero elasticity):

- $q_m^0 = q_m^1 = q$ and, hence, $P_L = \frac{\sum_{m=1}^{M} p_m^1 \cdot q}{\sum_{m=1}^{M} p_m^0 \cdot q} = \frac{\frac{1}{M}\sum_{m=1}^{M} p_m^1}{\frac{1}{M}\sum_{m=1}^{M} p_m^0} = P_D.$

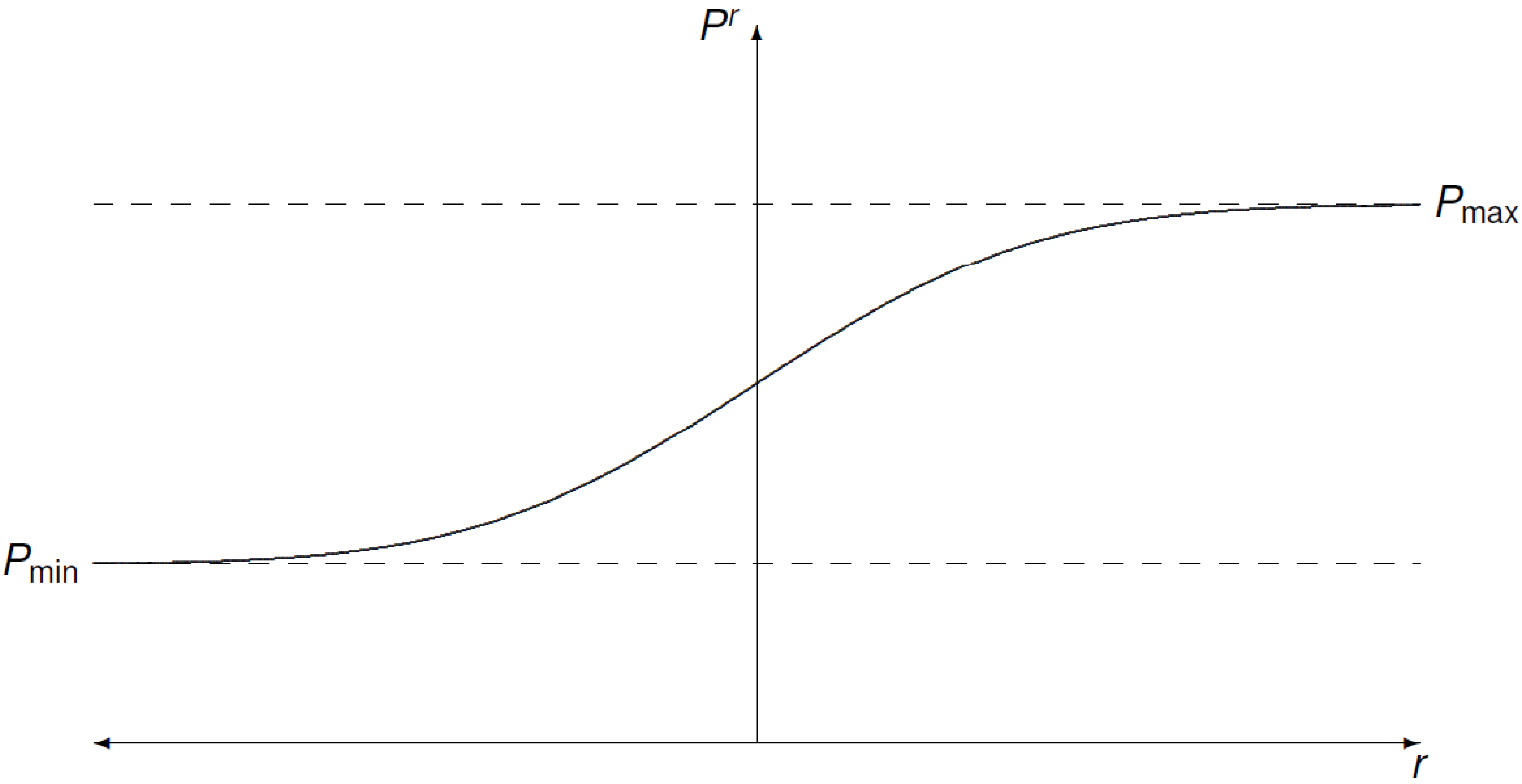
- The second case is when the preferences can be represented by **a Cobb-Douglas function**, i.e. **consumers vary the quantities** in inverse proportion to the changes in relative prices **so that expenditure shares remain constant** (unity elasticity):

- $s_i^0 = s_i^1 = M^{-1}$ and, hence, $P_L = \sum_{m=1}^{M} \frac{p_m^1}{p_m^0} \cdot M^{-1} = \frac{1}{M} \sum_{m=1}^{M} \frac{p_m^1}{p_m^0} = P_C.$

4. Consistency approach Generalised means

- A single comprehensive framework, known as *generalised means*, **unifies the aggregate and elementary levels**.
- **The generalised mean** of order r for the M item price ratios or price relatives, p_m^1/p_m^0, is defined as follows:
- $$P^r = \begin{cases} \sqrt[r]{\frac{1}{M}\sum_{m=1}^{M}\left(\frac{p_m^1}{p_m^0}\right)^r} & \text{if } r \neq 0, \\ \sqrt[M]{\prod_{m=1}^{M}\frac{p_m^1}{p_m^0}} & \text{if } r = 0. \end{cases}$$
- The generalised mean represents **a whole class of unweighted elementary indices**, such as the Carli and Jevons indices for r = 1 and r = 0, respectively.

4. Consistency approach Typical shape



$$P_{\text{min}} = \min\left(\left\{\frac{p_m^1}{p_m^0}\right\}\right),\ P_{\text{max}} = \max\left(\left\{\frac{p_m^1}{p_m^0}\right\}\right)$$

4. Consistency approach
CES aggregator function

- It is supposed that **the unit cost function** has the following functional form:

- $$c(\boldsymbol{p}) = \begin{cases} \alpha_0 \cdot \left(\sum_{m=1}^{M} \alpha_m \cdot p_m^{1-\sigma} \right)^{1/(1-\sigma)} & \text{if } \sigma \neq 1, \\ \alpha_0 \cdot \prod_{m=1}^{M} p_m^{\alpha_m} & \text{if } \sigma = 1, \end{cases}$$

- where the α_m are non-negative consumer preference parameters with $\sum_{m=1}^{M} \alpha_m = 1$.
- This unit cost function corresponds to **a CES aggregator or utility function**.
- The parameter σ is **the elasticity of substitution**:
  - When $\sigma = 0$, the underlying preferences are **Leontief preferences**.
  - When $\sigma = 1$, the corresponding utility function is **a Cobb-Douglas function**.

4. Consistency approach
Laspeyres and Paasche price indices

- A generalised mean of order *r* **equal to the elasticity of substitution** (σ) yields approximately the same result as **the Laspeyres price index**.
- Hence, if the elasticity of substitution is one (Cobb-Douglas preferences), for example, *r* must equal one and **the Carli index at the elementary level will correspond to the Laspeyres price index as target index**.

- However, if **the Paasche price index** should be replicated, the order of the generalised mean must **equal minus the elasticity of substitution**, in the above example minus one.
- Thus, **the harmonic index gives the same result** and therefore, in this case it should be used at the elementary level.

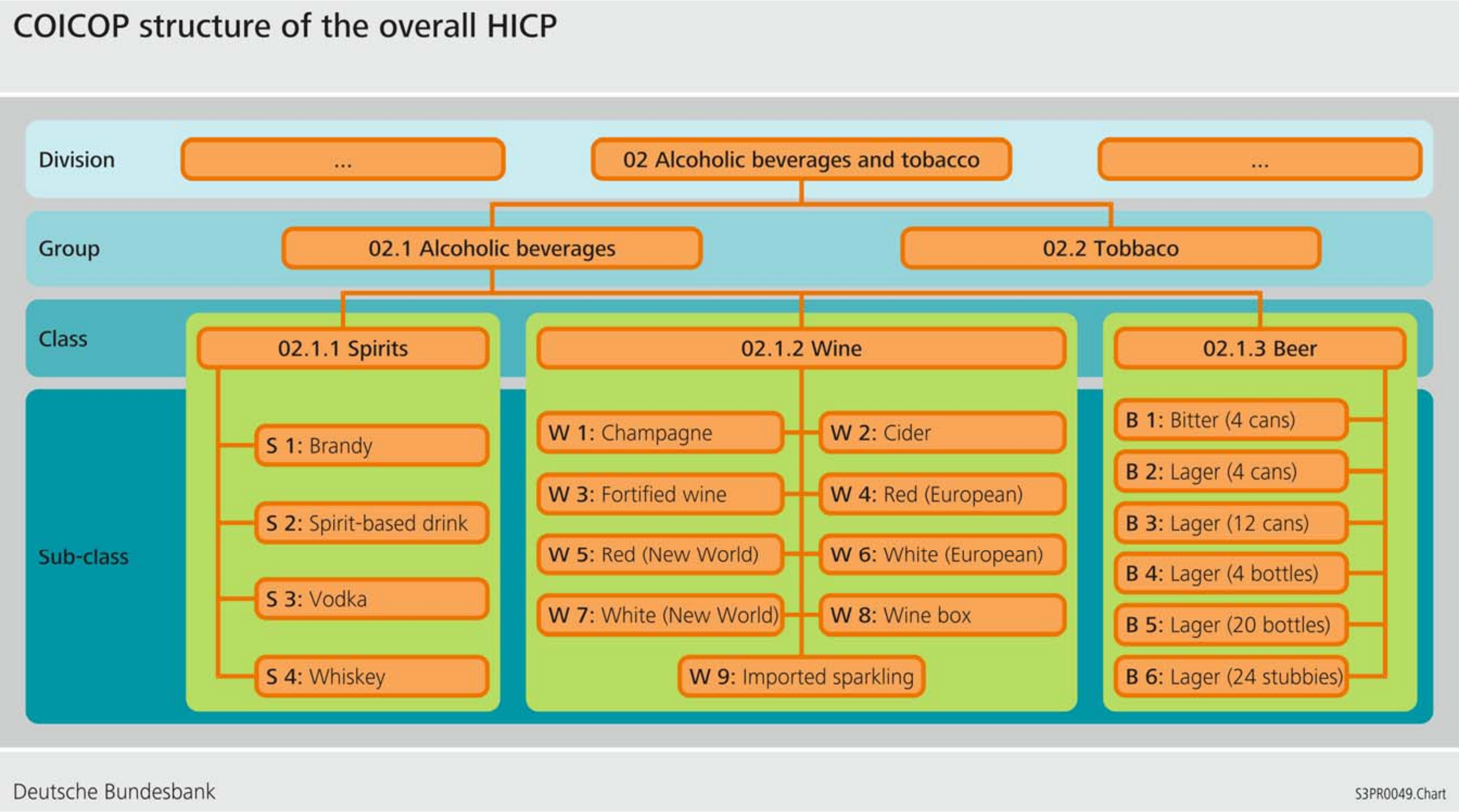
- **Only if the elasticity of substitution is zero** (Leontief preferences), **the Jevons (Dutot) index corresponds to both the Laspeyres and Paasche price indices** – which in this case coincide.

5. Empirical results
Alcoholic beverages in the UK

- As an empirical application, **detailed expenditure data** from Kantar Worldpanel for elementary aggregates within the COICOP group of **alcoholic beverages in the UK** are analysed.
- The data cover the period **from January 2003 to December 2011**; the data set consists of transaction level data, which records inter alia purchase price and quantity, and includes **192,948 observations** after outlier identification.

- **The elasticity of substitution** is estimated in the framework of a log-linear model by means of **ordinary least squares**. (Note that the consumer preference parameters are removed via differencing products common to adjacent months and, thus, there is no need for application of seemingly unrelated regression.)
- As a robustness check to the CES model based results, **the generalised mean** which minimises relative bias and root mean squared relative error to the desired aggregate index is found directly by **numerical optimisation techniques**. (Rather than at the aggregate transaction level, like the econometric method, this analysis, however, is performed one level above – at the elementary level.)

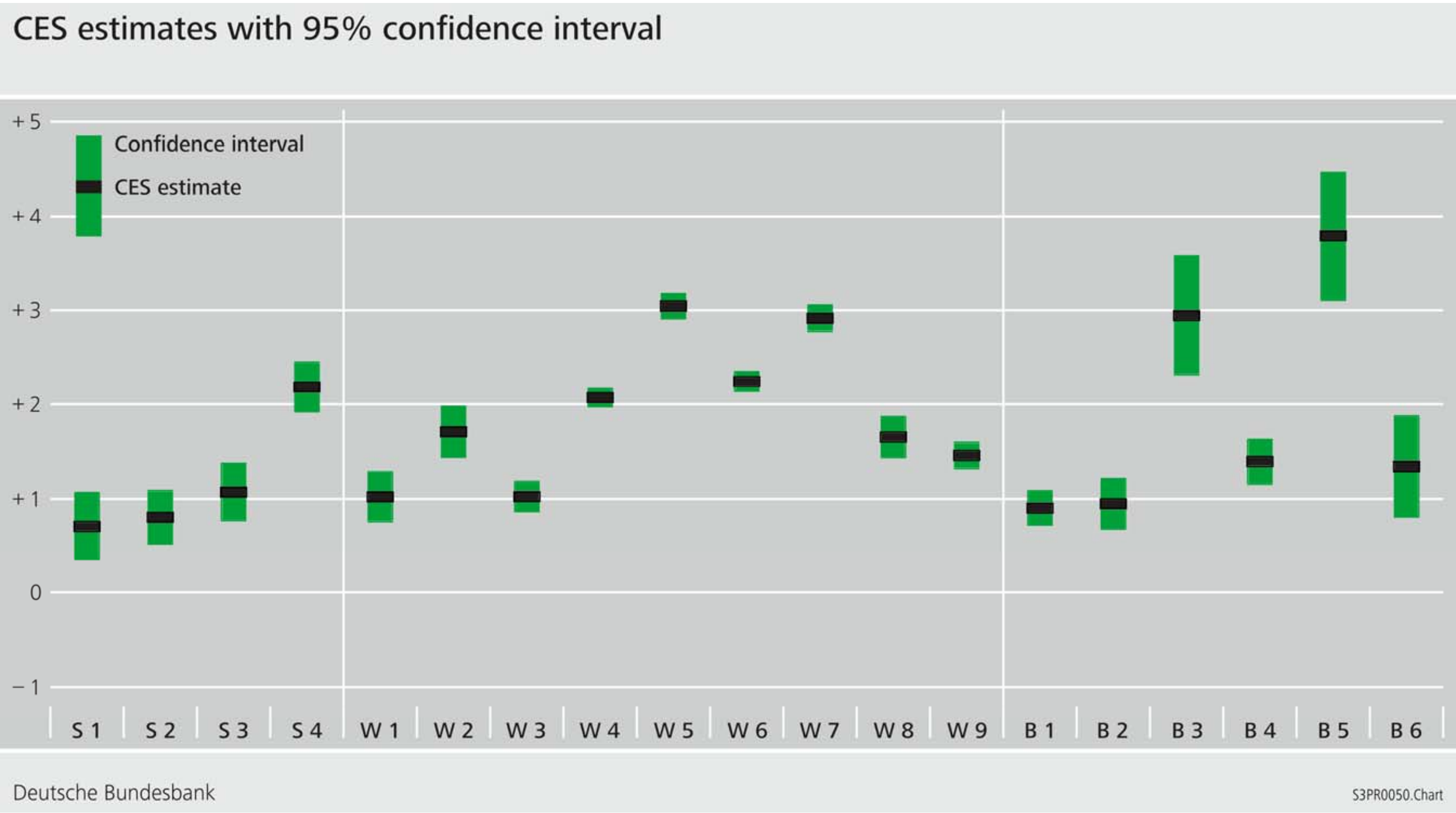
5. Empirical results
COICOP structure for alcoholic beverages



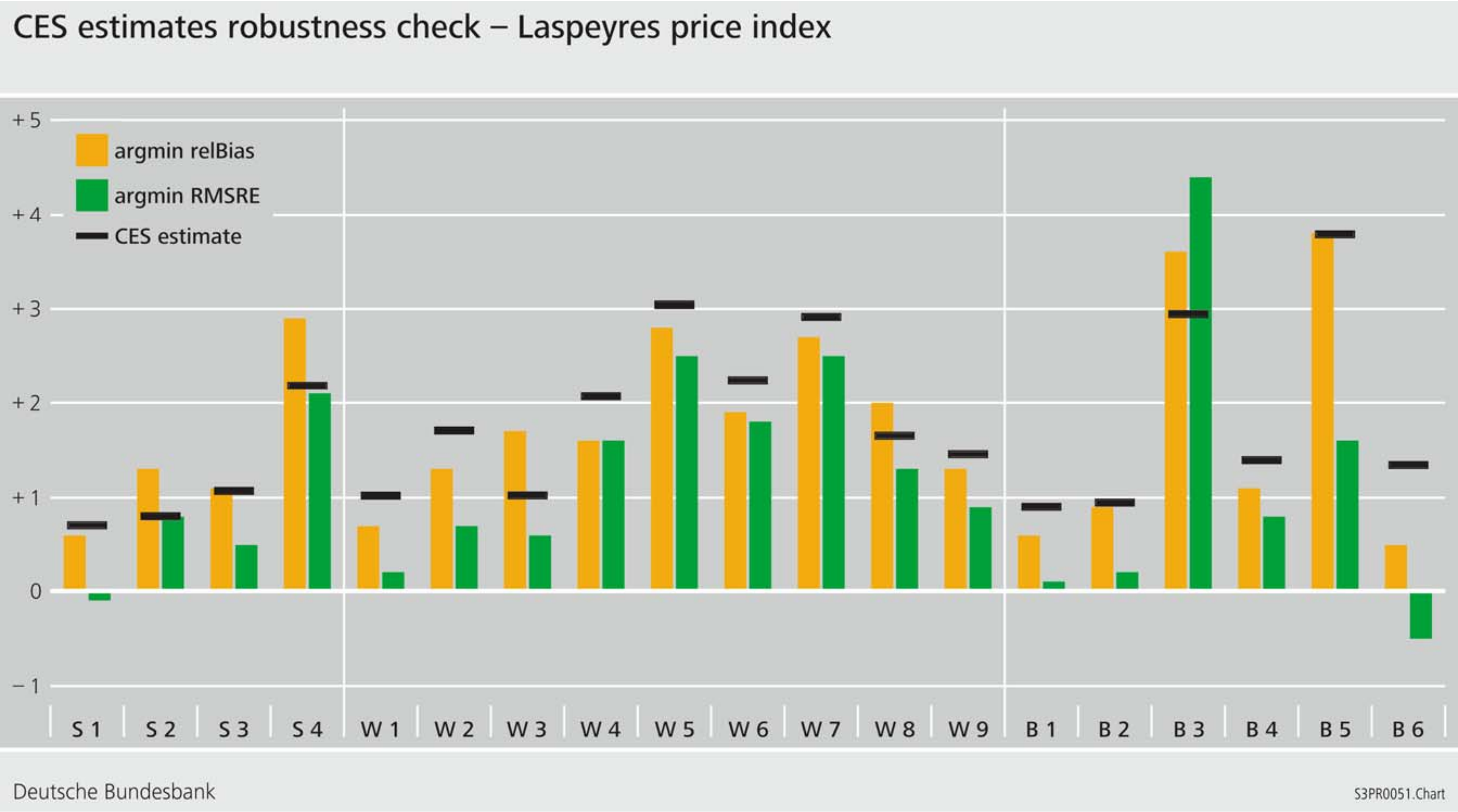
5. Empirical results
Findings on substitution behaviour

- The median elasticity of substitution is 1.5, **ranging from .7 to 3.8**.
- All estimates are **statistically significantly greater than zero**; for 8 out of 19 sub-classes **the difference to iso-elasticity is insignificant**, while for the remaining 11 sub-classes substitution is found to even exceed unity elasticity.
- **In spirits, consumers are more willing to substitute between different types of whiskey (S 4) than is the case for brandy or vodka (S 1 and S 2).**
- **For both red and white wines, substitution is more pronounced for the New World (W 5 and W 7) than for European wines (W 4 and W 6).**
- **Also, the elasticity of substitution tends to be higher for 12 cans and 20 bottles of lager (B 3 and B 5), respectively, than for 4 packs (B 2 and B 4).**
- These results are **consistent with the findings of Elliott and O’Neill (2012)**.
- Furthermore, comparing the CES regression results with the direct calculation of the generalised means, **the outcomes do not change qualitatively**.
- In particular, **the Carli index performs remarkably well** at the elementary level of a Laspeyres price index, questioning the argument of its “upward bias” – in fact, **it is the Jevons index that has a downward bias**.

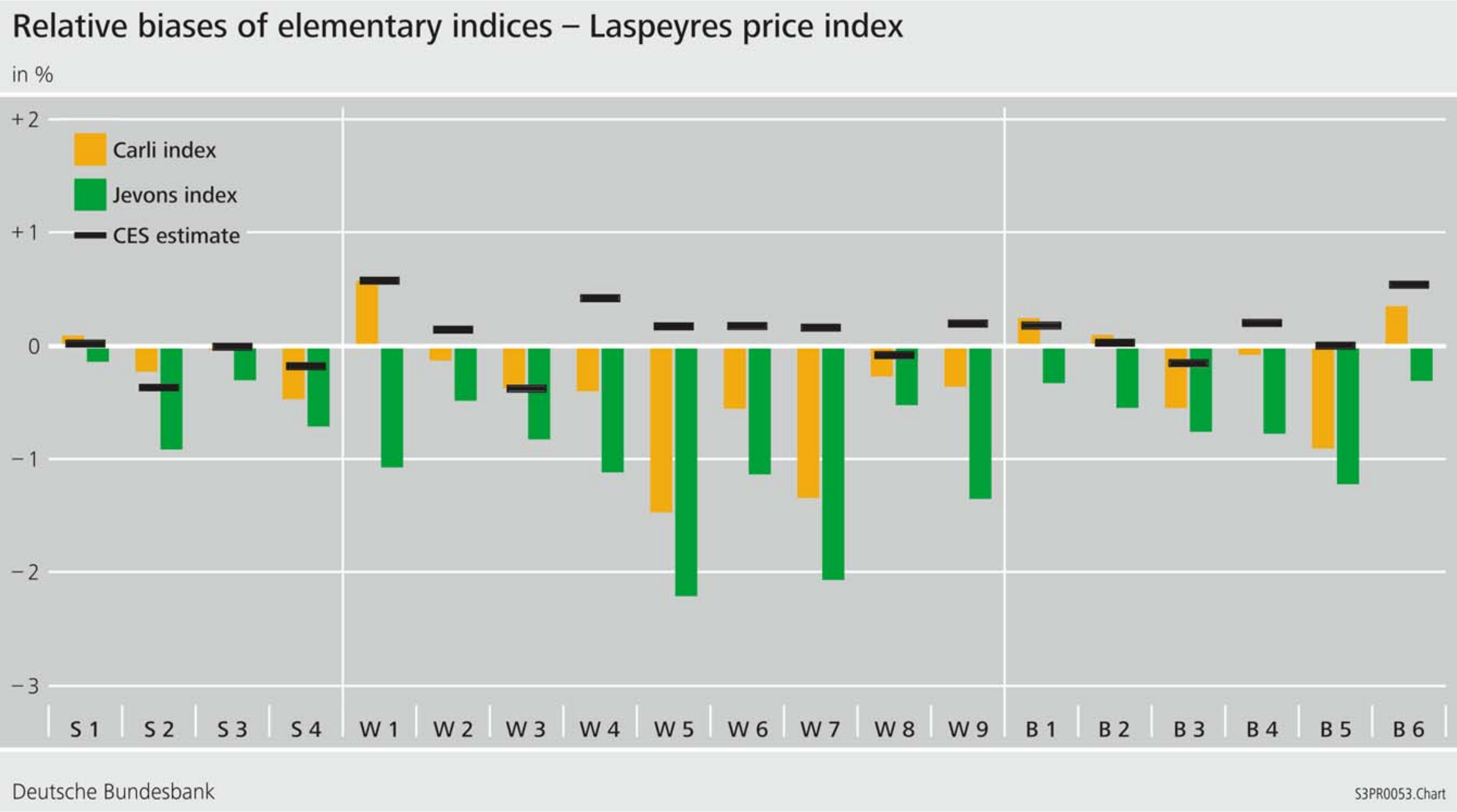
5. Empirical results
CES estimation results



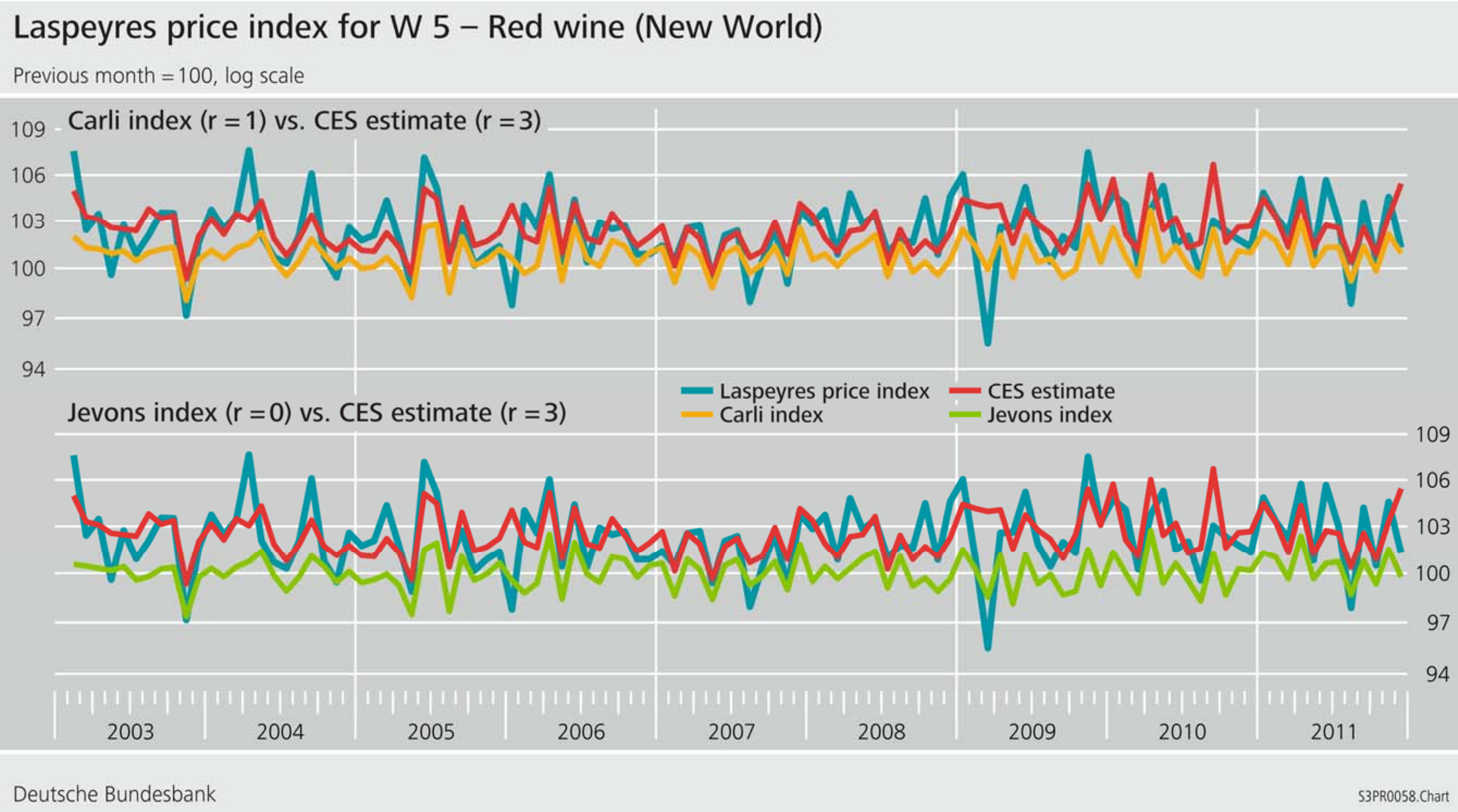
5. Empirical results
Laspeyres price index: robustness



5. Empirical results
Laspeyres price index: bias



5. Empirical results
Laspeyres price index: time series



6. Discussion

- The problem of aggregational consistency demonstrates the need for **a weighting at the lowest possible level**.
- This would mean that, in the trade-off between **estimated weights/weights from secondary sources** on the one hand and the elementary bias of unweighted indices on the other, the balance would often tip **in favour of weighting**.
- **The biases at the elementary level** can, in some cases, reach such large dimensions that they become **relevant for the aggregate index**.
- There is **a "price" to be paid at the upper level** for suboptimal index formula selection at the lower level; thus, the need for **two-staged price indices to be accurately constructed** becomes obvious.
- **Disaggregation is a panacea!**

- Insofar as **no information on weights** is available, **studies on substitution** can help in **guiding the choice of the optimal elementary index** for a given measurement target.
- Often, even an expert judgement on substitutability **outperforms the test approach**.